



SCALABLE DISTRIBUTED SUBGRAPH ENUMERATION

AUTHORS: *LONGBIN LAI*
 LU QIN
 XUEMIN LIN
 YING ZHANG
 LIJUN CHANG

OUTLINE

PROBLEM DEFINITION

ALGORITHM FRAMEWORK

TWINTWIG JOIN - VLDB15'

SEED

EXPERIMENTS

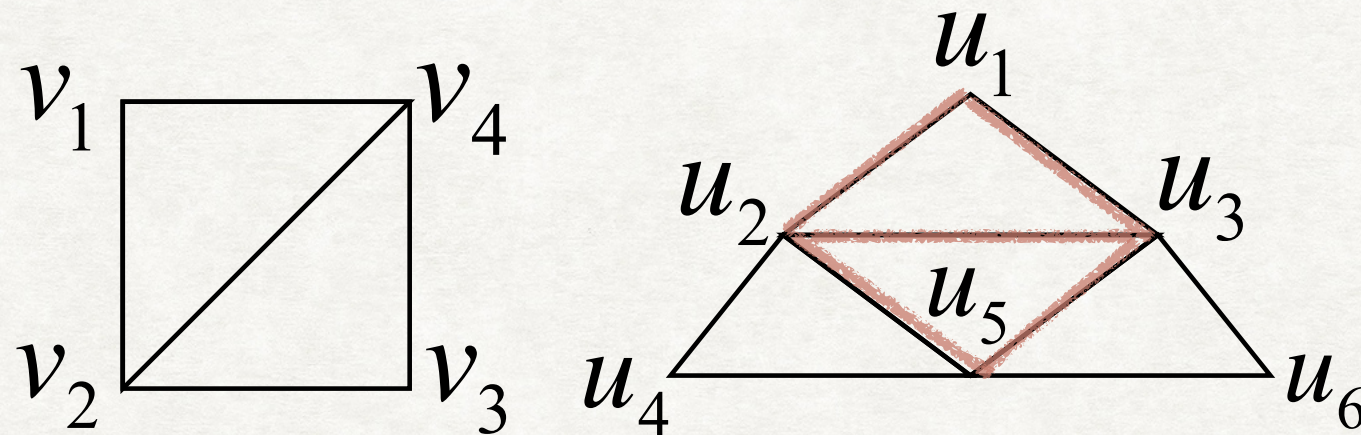
CONCLUSION

PROBLEM

PROBLEM DEFINITION

SUBGRAPH ENUMERATION

- Given a data graph G , and a pattern graph P , subgraph enumeration aims to find all subgraphs $g \subseteq G$ (matches), that are isomorphic to P .



P

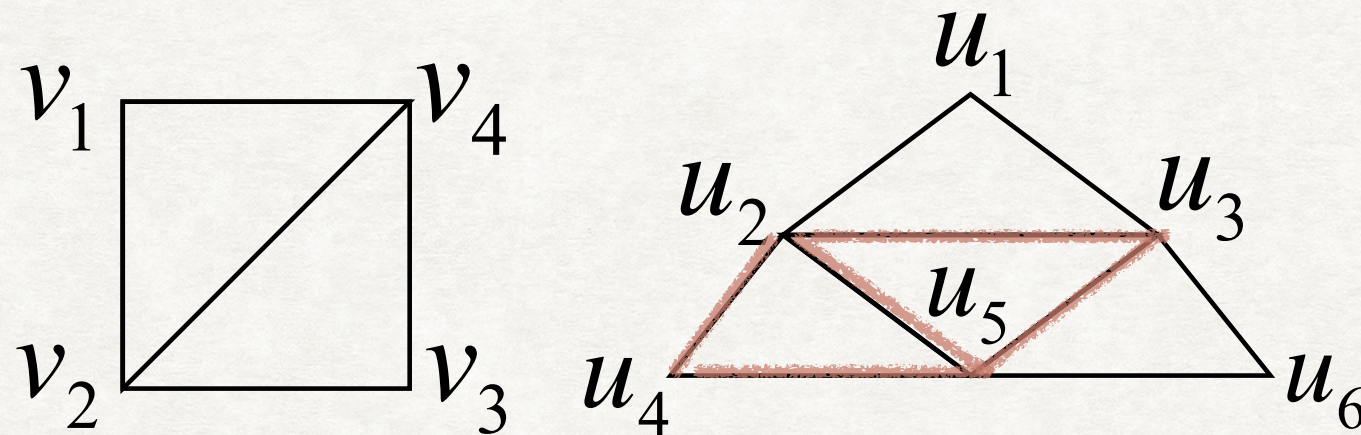
G

$$\begin{pmatrix} v_1 & v_2 & v_3 & v_4 \\ u_1 & u_2 & u_5 & u_3 \end{pmatrix}$$

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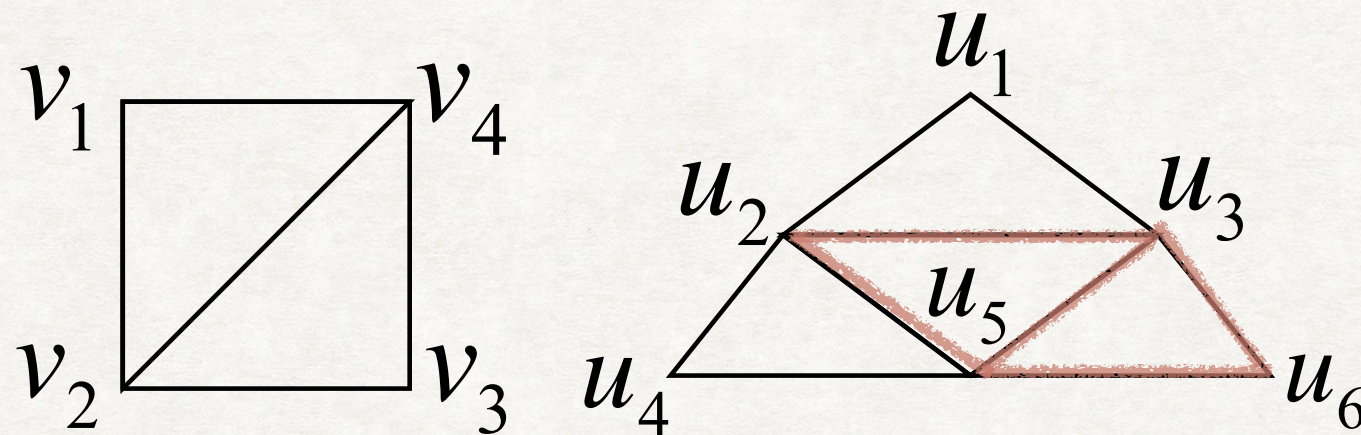
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$$\begin{pmatrix} v_1 & v_2 & v_3 & v_4 \\ u_4 & u_2 & u_3 & u_5 \end{pmatrix}$$

PROBLEM DEFINITION

SUBGRAPH ENUMERATION

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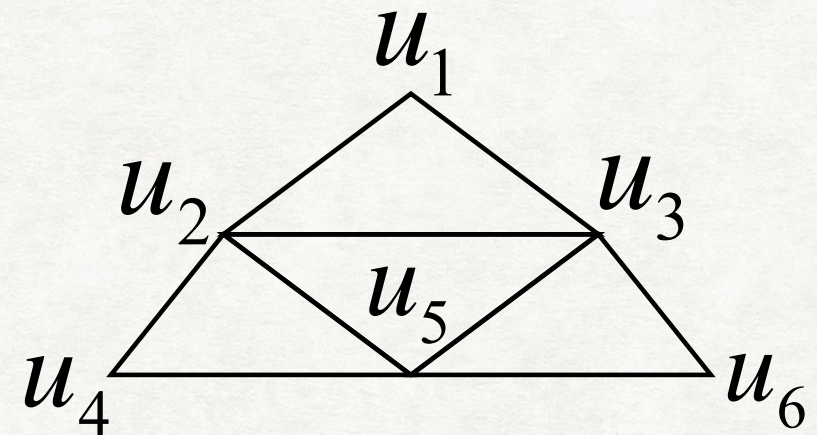
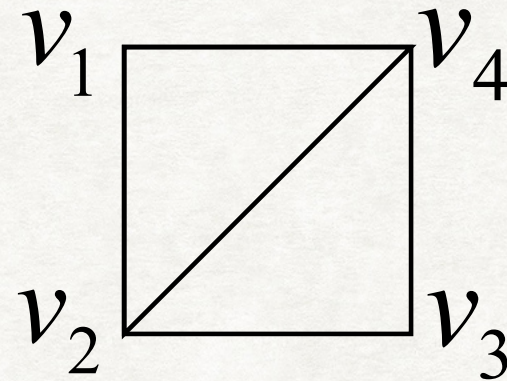
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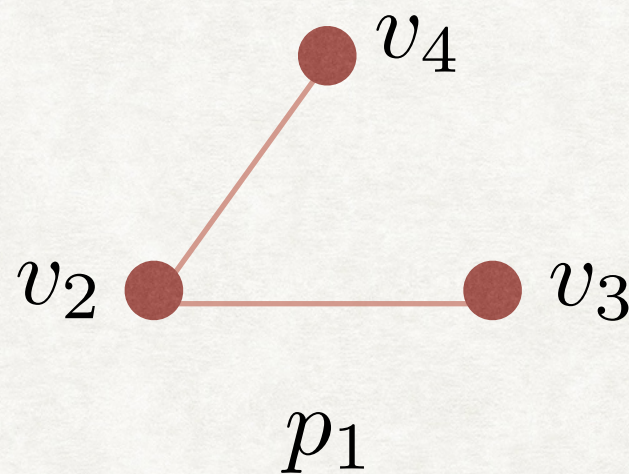
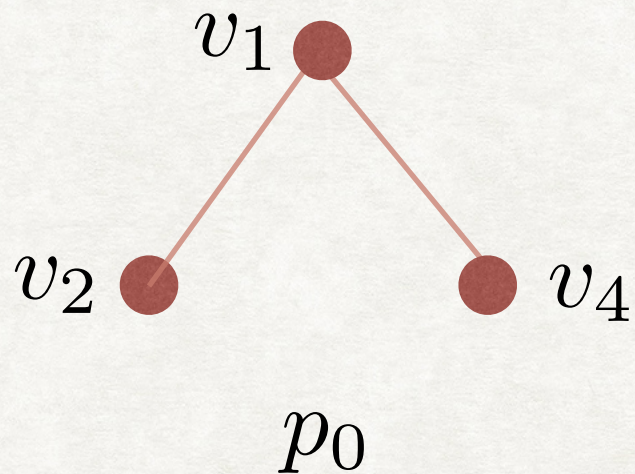
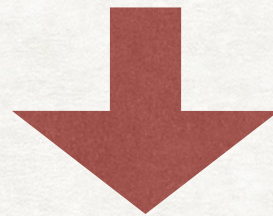
$$\begin{pmatrix} v_1 & v_2 & v_3 & v_4 \\ u_6 & u_3 & u_2 & u_5 \end{pmatrix}$$

FRAMEWORK

PATTERN DECOMPOSITION



$$P = p_0 \cup p_1 \cup p_2$$



Join Units

WHAT CAN BE JOIN UNITS

- Graph Storage $\Phi(G) = \{G_u | u \in V(G)\}$
 - Stored as $(u; G_u)$ for each data node
 - G_u : Local Graph of u s.t.
 - (1) Connected
 - (2) $u \in V(G_u)$
 - (3) $\bigcup_{u \in V(G)} E(G_u) = E(G)$

WHAT CAN BE JOIN UNITS

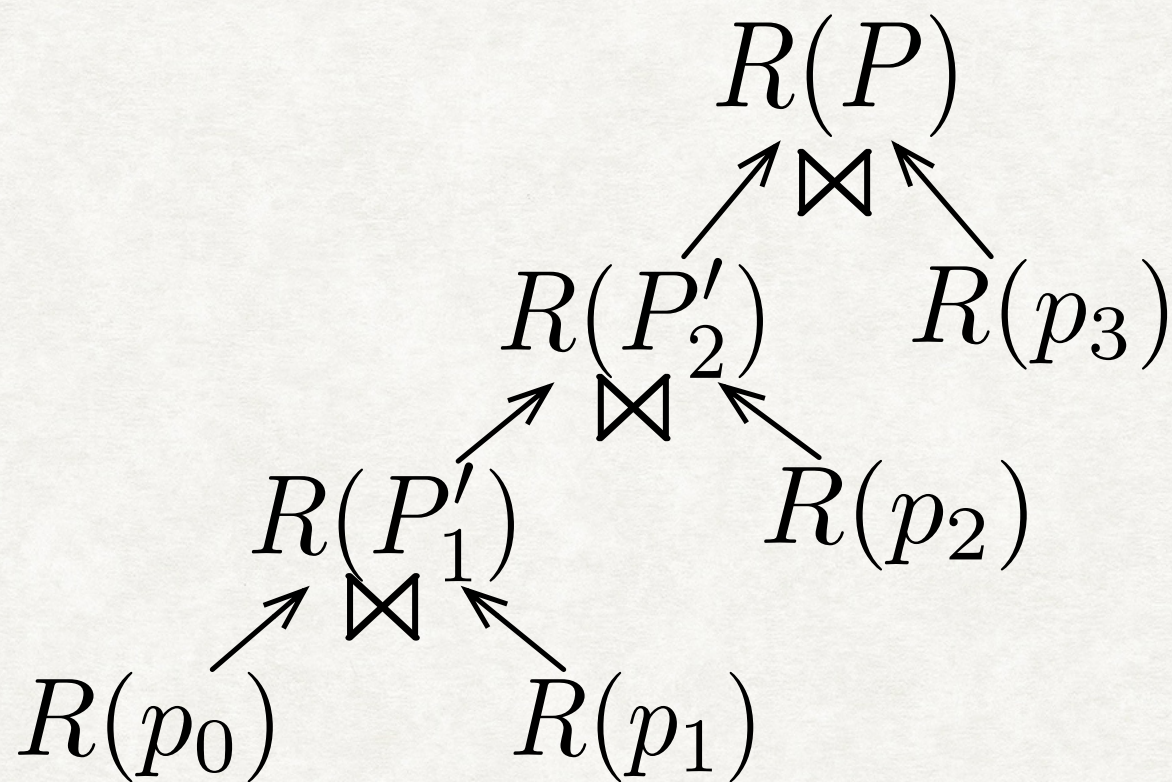
- A structure p can be a join unit iff.

$$R_G(p) = \bigcup_{u \in V(G)} R_{G_u}(p)$$

- $R_G(p)$ stands for the matches of p in \mathcal{G}

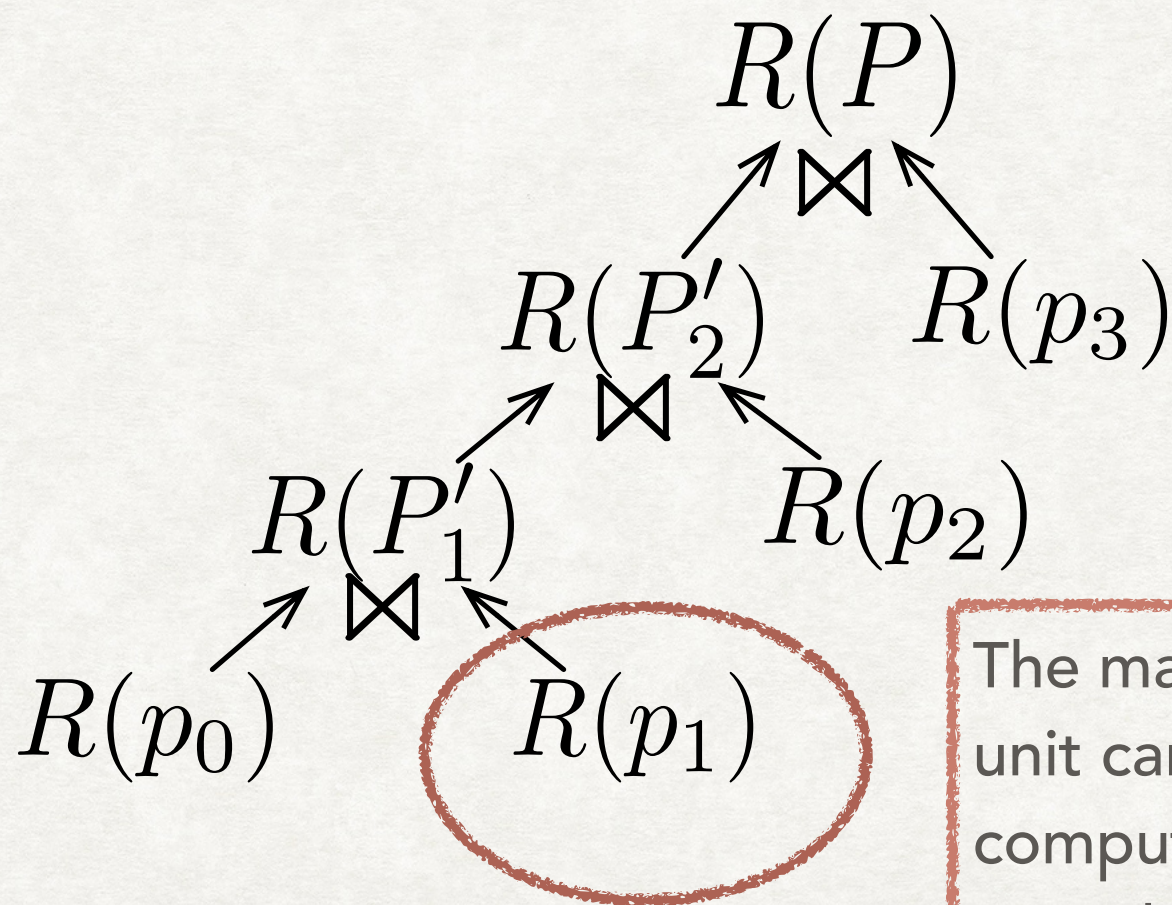
JOIN PLAN (TREE)

- Decomposing $P = p_0 \cup p_1 \cup p_2 \cup p_3$
- Solving: $R(P) = R(p_0) \bowtie R(p_1) \bowtie R(p_2) \bowtie R(p_3)$



JOIN PLAN (TREE)

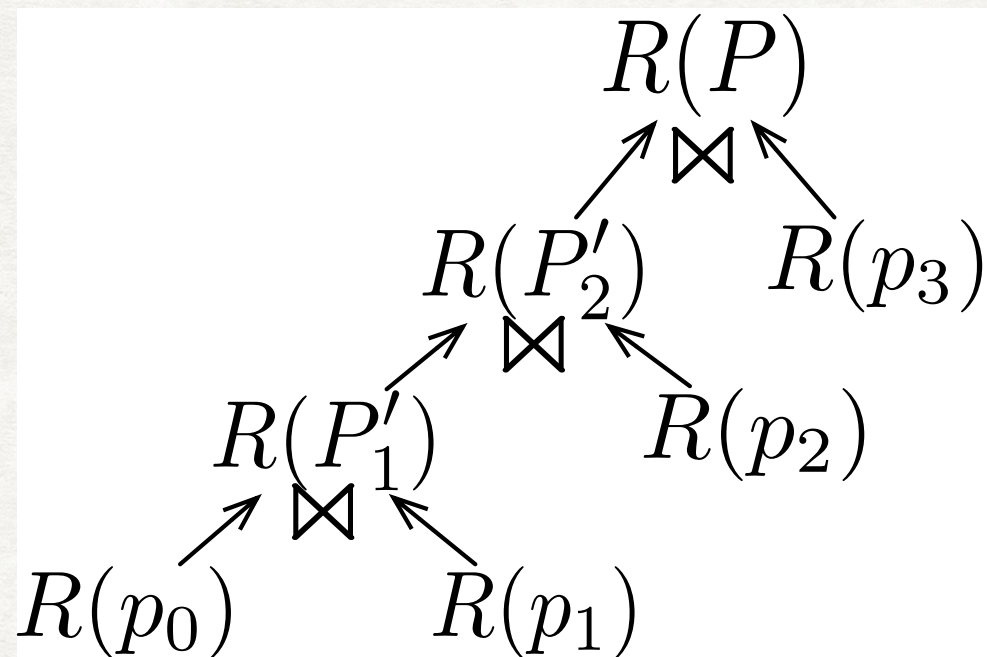
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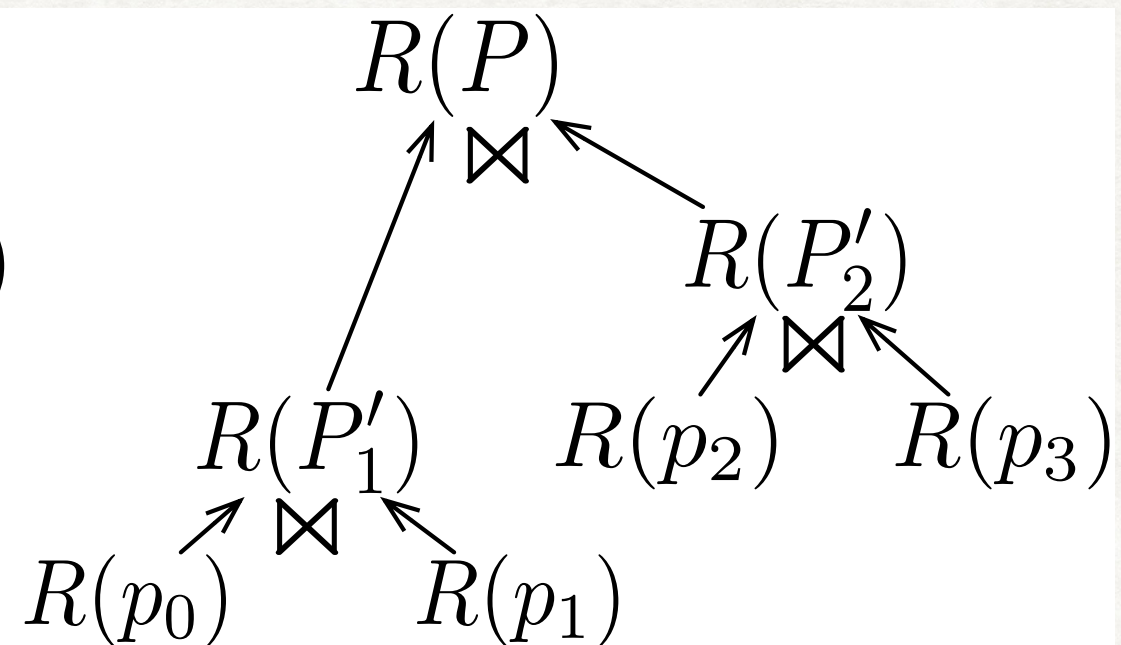
The matches of each join unit can be online computed independently in each local graph

JOIN PLAN (TREE)

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Left-deep tree



Bushy tree

DESCRIBE THE ALGORITHMS

- Graph Storage mechanism
 - Determine the join units, thereafter the pattern decomposition
- Join Structure
 - Left-deep tree vs bushy tree

TWINTWIG
JOIN - VLDB15'

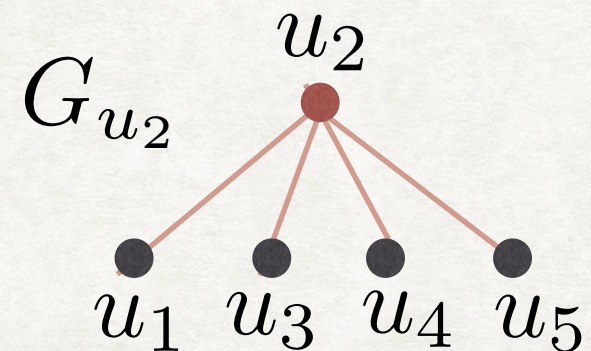
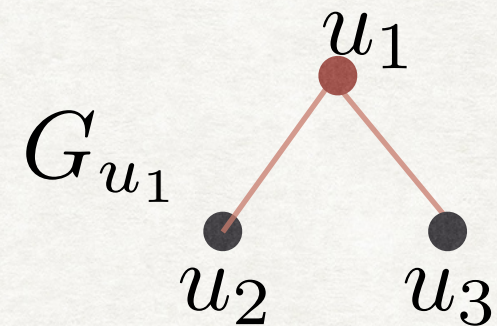
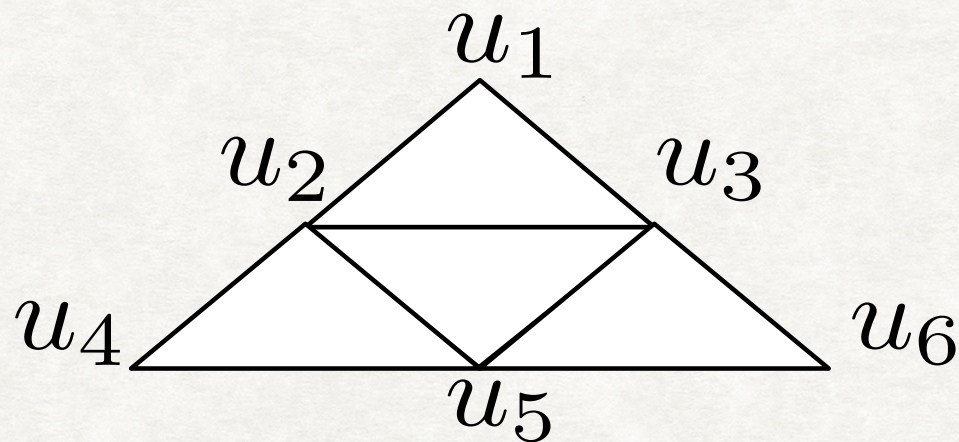
TWINTWIG JOIN - VLDB2015

SIMPLE GRAPH STORAGE

- The simple graph storage, each local graph G_u

$$V(G_u) = \{u\} \cup \mathcal{N}(u)$$

$$E(G_u) = \{(u, u') \mid u' \in \mathcal{N}(u)\}$$



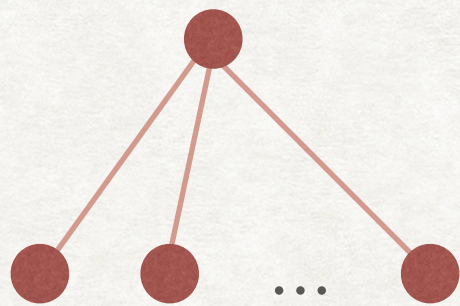
TWINTWIG JOIN - VLDB2015

SIMPLE GRAPH STORAGE

- The simple graph storage, where

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Star as the join unit

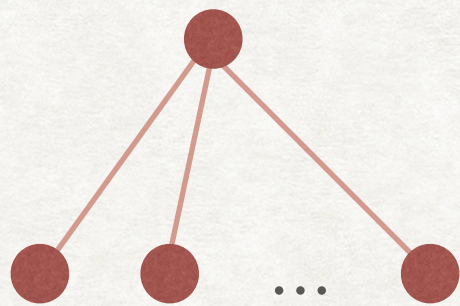
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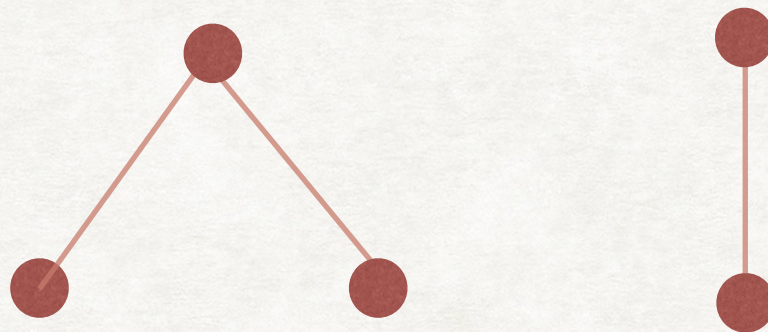
Star as the join unit

A node with degree 1,000,000
will generate 10^{18} 3-stars

TWINTWIG JOIN

SIMPLE GRAPH STORAGE

- Using twintwigs as the join units

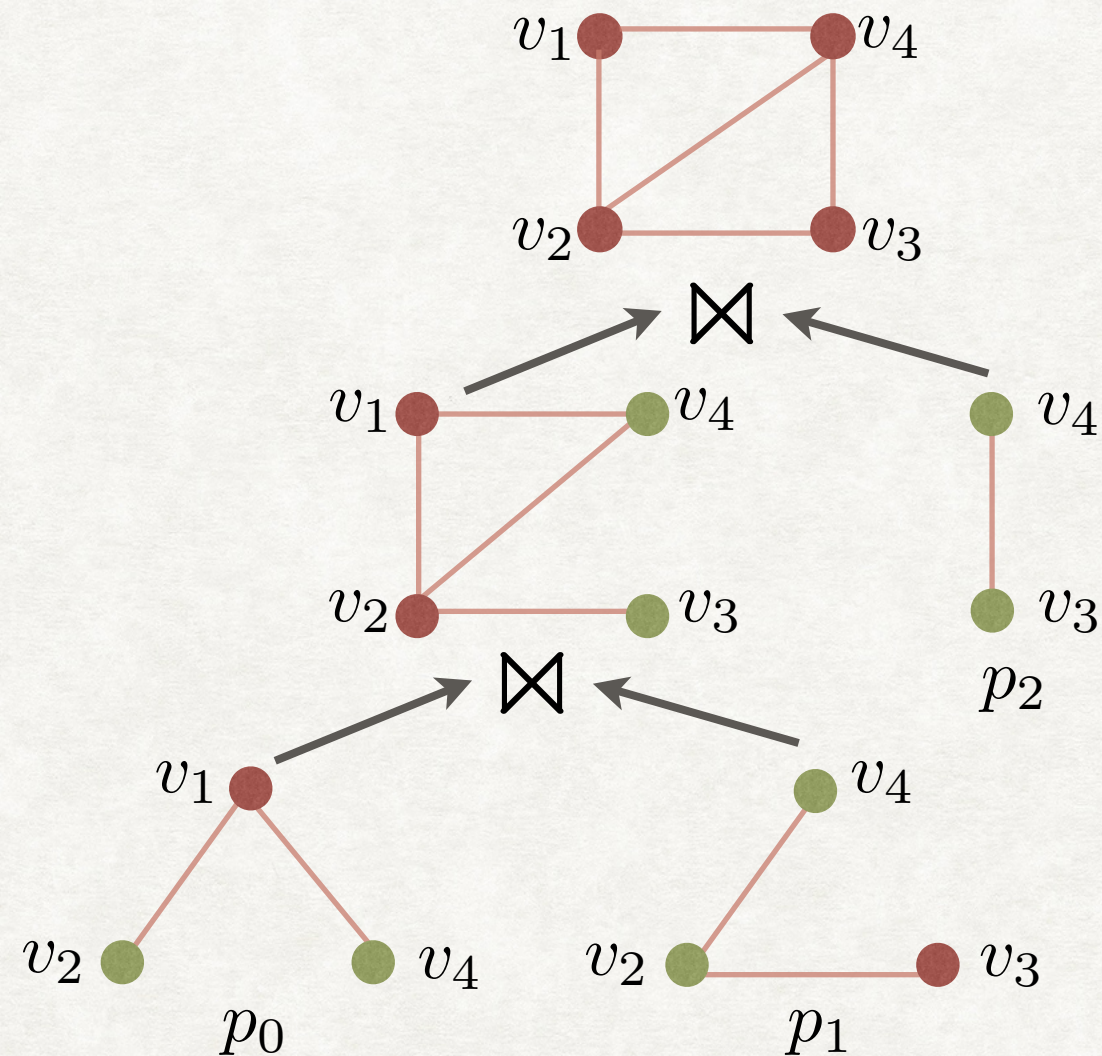


- Instance Optimality
 - Given any join plan involving general stars, we can solve it using twintwigs with at most the same (**often much less**) cost

TWINTWIG JOIN

LEFT-DEEP JOIN PLAN

- An optimal left-deep join plan with minimum estimated cost



TWINTWIG JOIN

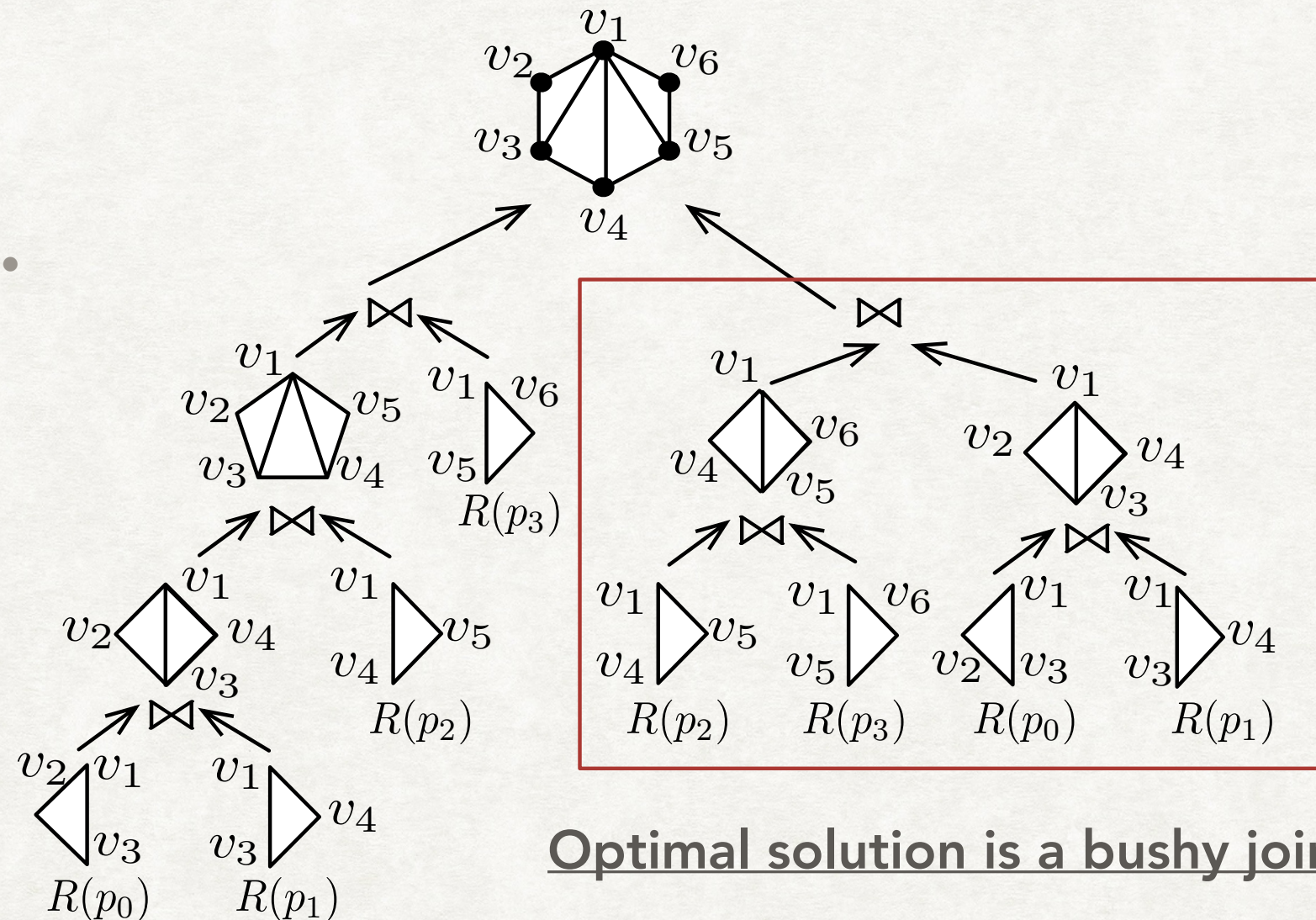
DRAWBACKS

- Simple storage mechanism only support using star as join units, too many intermediate results
- Twintwig: confine to be at most two edges
 - The node with degree 1,000,000 still have 10^{12} two-edge twintwigs
- Too many execution rounds.
 - A clique of 6 nodes (15 edges): Seven rounds of TwinTwigJoin

TWINTWIG JOIN

DRAWBACKS

- Left-deep join: may result in sub-optimal results



Optimal solution is a bushy join

SEED - VLDB17'

MOTIVATIONS

- Subgraph EnumEration in Distributed Context
 - SCP (Star-Clique-Preserved) graph storage: Use star and clique as the join units
 - We can avoid using star if clique is an alternative
 - Shorter execution. The 6-clique can now be processed in one single round, instead of 7 rounds in TwinTwigJoin
 - Bushy join plan: Optimality Guarantee
 - Much better performance

SEED

SEED

SCP GRAPH STORAGE

- The SCP Graph Storage, where each local graph G_u^+

$$V(G_u^+) = V(G_u) = \{u\} \cup \mathcal{N}(u)$$

$$E(G_u^+) = E(G_u) \cup \{(u', u'') \mid (u', u'') \in E(G) \wedge u', u'' \in \mathcal{N}(u)\}$$

SEED

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NEIGHBOUR EDGES

SEED

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TRIANGLE EDGES

SEED

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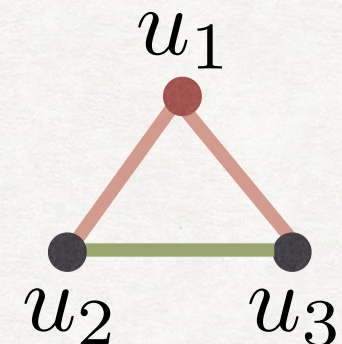
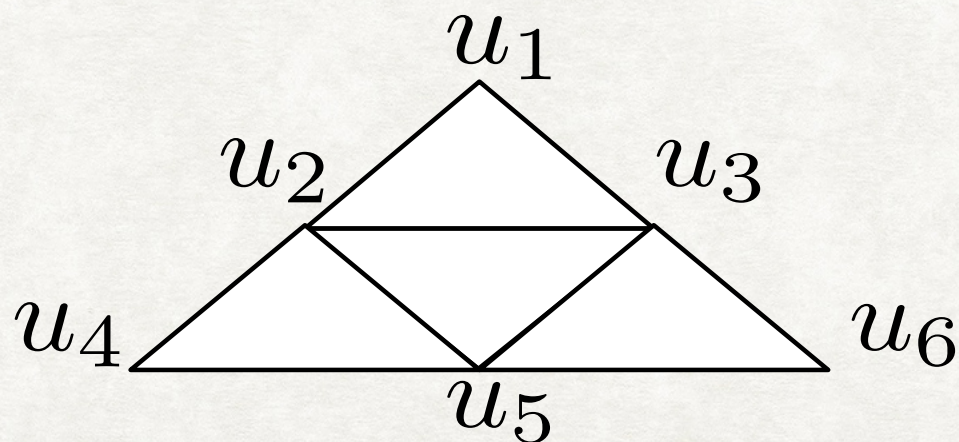
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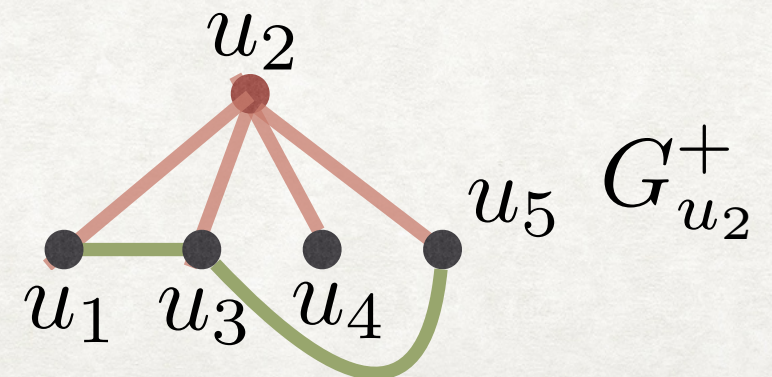
$$\{(u', u'') \mid (u', u'') \in E(G) \wedge u', u'' \in \mathcal{N}(u)\}$$

NEIGHBOUR EDGES

TRIANGLE EDGES



$G_{u_1}^+$



$G_{u_2}^+$

SEED

SCP GRAPH STORAGE

- We show that SCP graph storage supports using both star and clique as the join units
- A more compact version which has bounded size for each local graph

SEED

OPTIMAL BUSHY JOIN PLAN

- Notations
 - E_P : The join plan to solve P
 - $C(E_P)$: The cost of the join plan
 - $C(P)$: Estimated # matches of P in G
- We aim at finding a join plan for P , s.t.

$C(E_P)$ is minimised

SEED

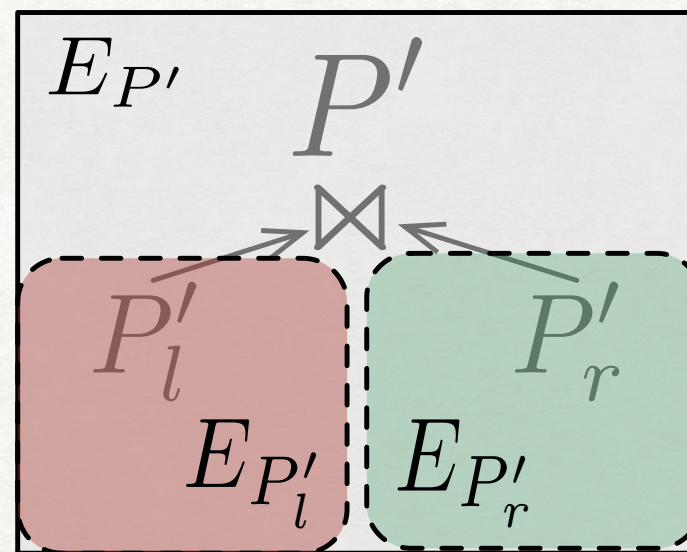
OPTIMAL BUSHY JOIN PLAN

- A dynamic programming transform function

- e.g. $E_{P'}$

- (1) $E_{P'_l}$

- (2) $E_{P'_r}$



- (3) $R(P') = R(P'_l) \bowtie R(P'_r)$

SEED

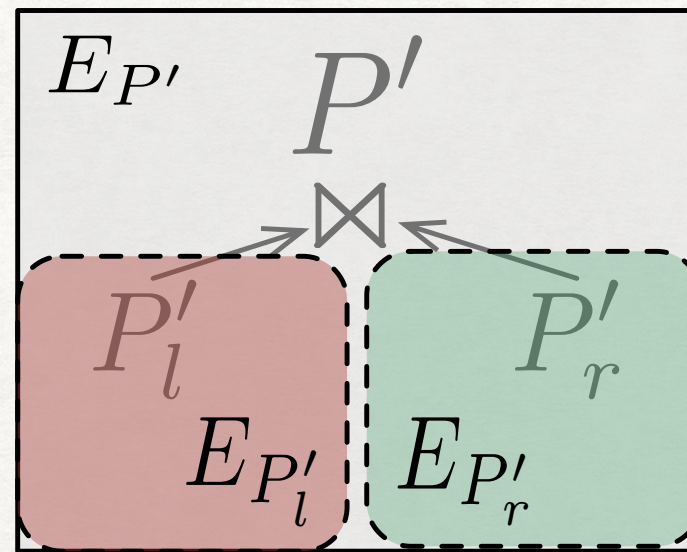
OPTIMAL BUSHY JOIN PLAN

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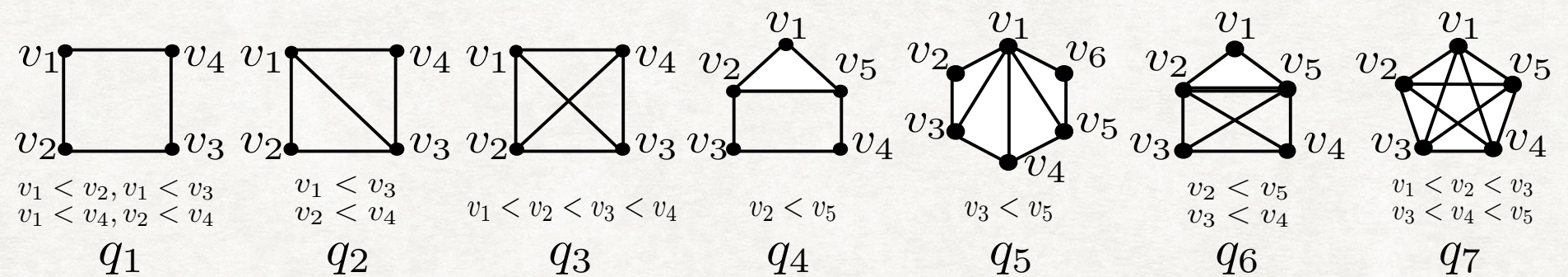
$$C(E_{P'}) = \min_{P'_l \subset P' \wedge P'_r = P' \setminus P'_l} \{C(E_{P'_l}) + C(P'_l) + C(E_{P'_r}) + C(P'_r)\}$$

EXPERIMENTS

EXPERIMENTS

SETUP

- Queries



- Algorithms

- SEED+O (The most optimised SEED)
- TT (The most optimised TwinTwigJoin, VLDB 2015)
- pSgL (Shao et al. Sigmod 2014)

EXPERIMENTS

SETUP

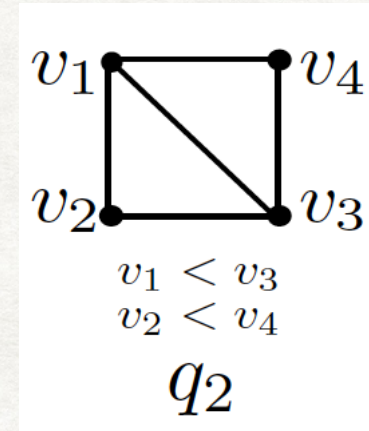
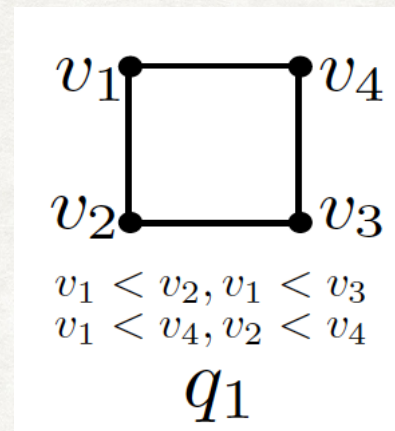
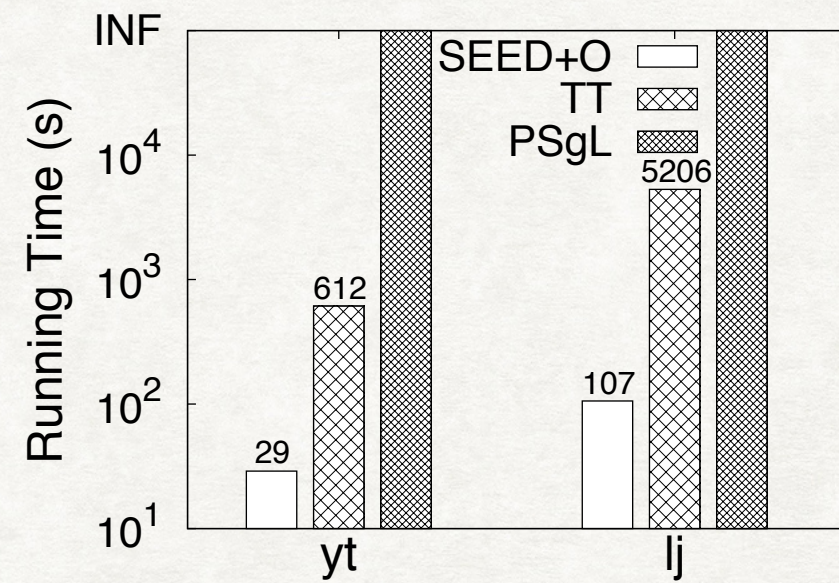
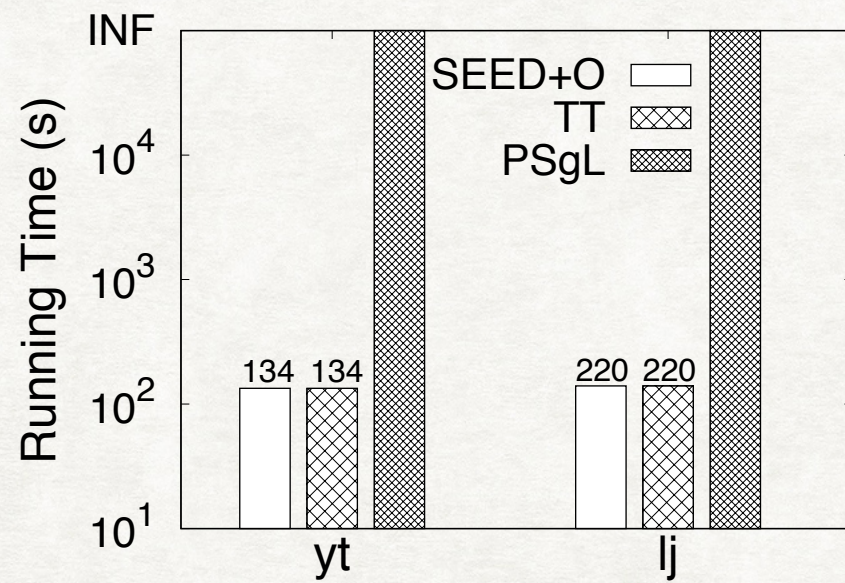
- Cluster
 - Amazon EC2: 1 master node, 10 slave nodes

Node	Instance	vCPU	Memory	Disk
master	m3.xlarge	4	15GB	2 x 40GBSSD
slave	c3.4xlarge	16	30GB	2 x 160GB SSD

- Hadoop 2.6.2
 - JVM heap space: mapper 1524MB, reducer 2848MB
 - 6 mappers and 6 reducers each machine

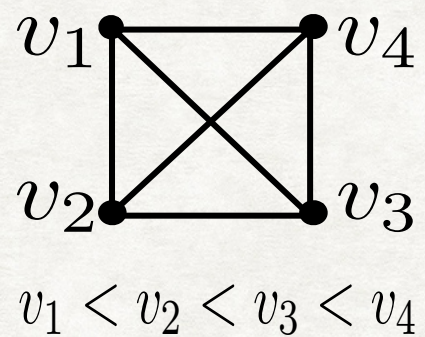
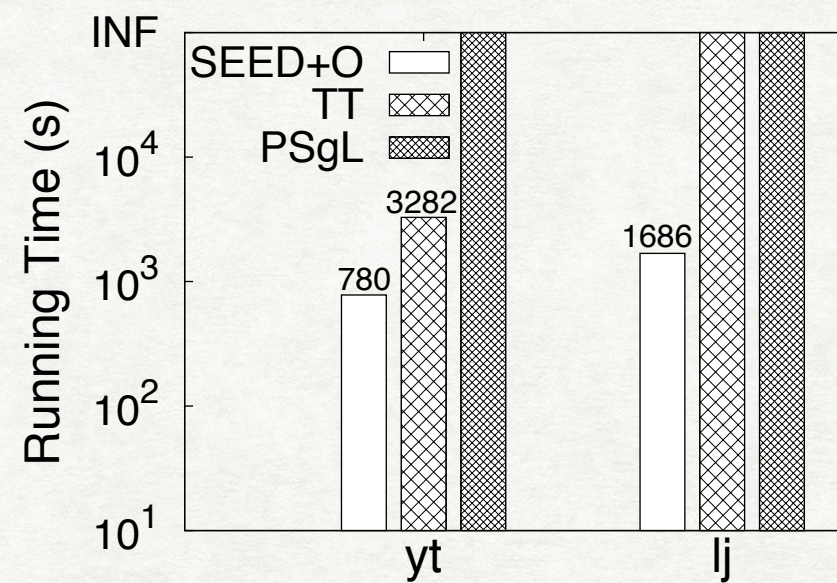
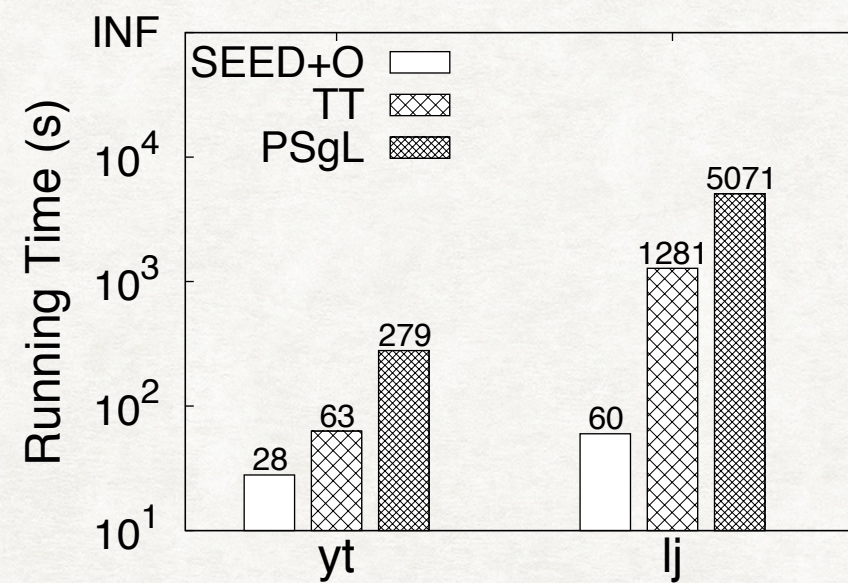
EXPERIMENTS

RESULTS

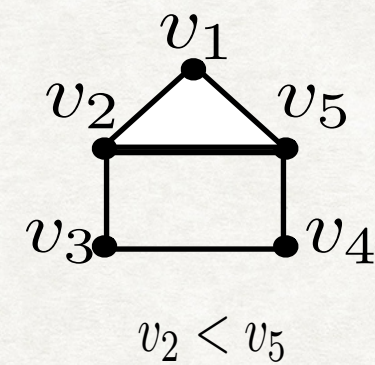


EXPERIMENTS

RESULTS



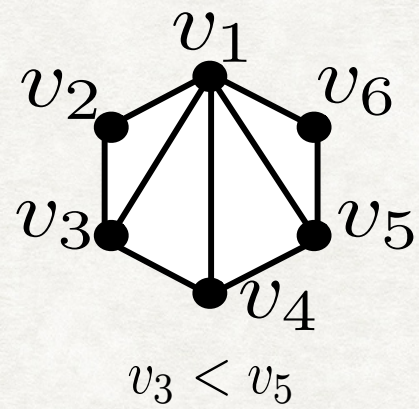
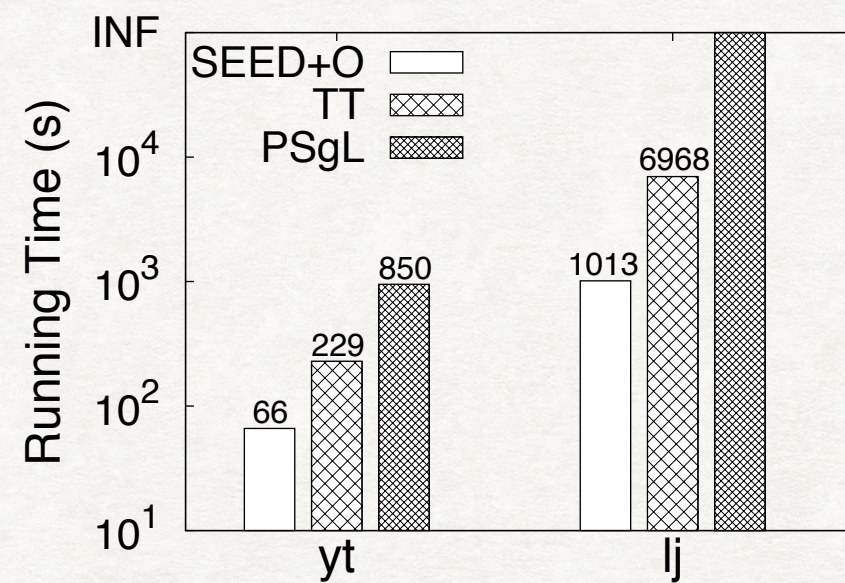
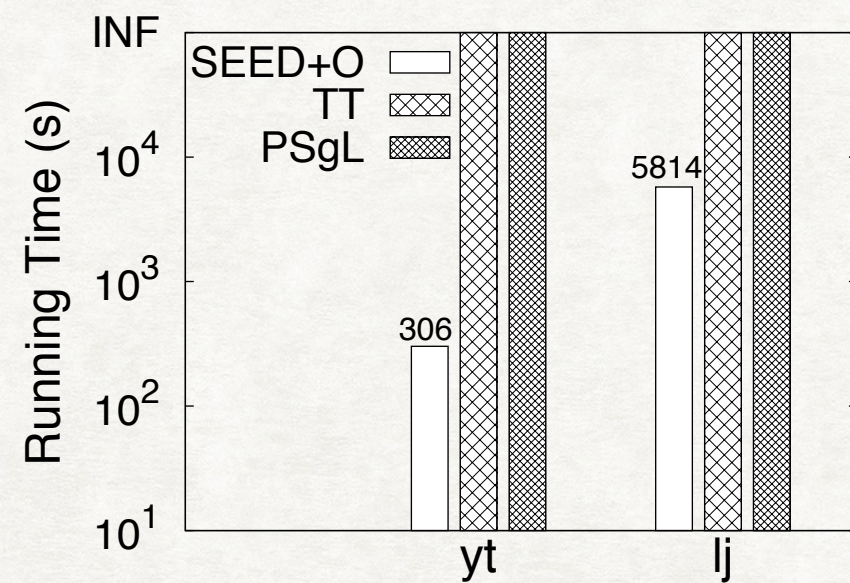
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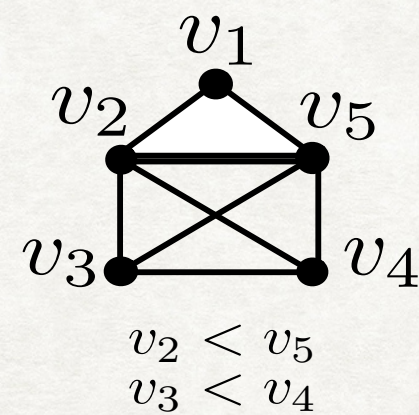
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EXPERIMENTS

RESULTS



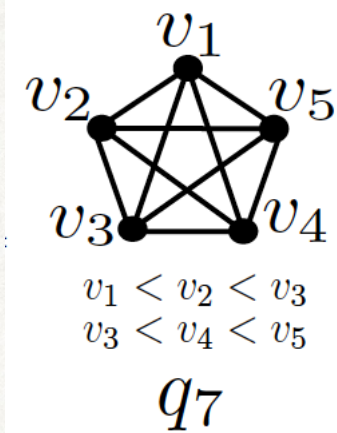
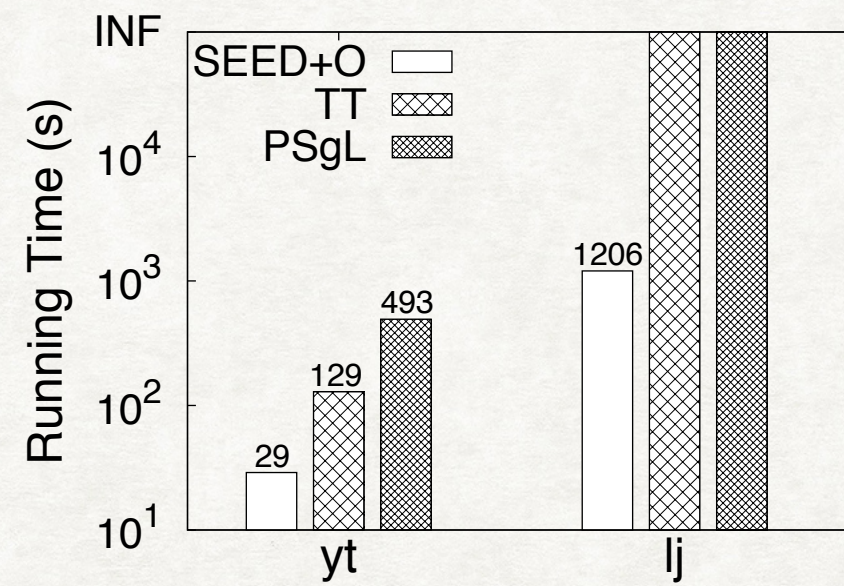
q_5



q_6

EXPERIMENTS

RESULTS



CONCLUSION

- A general decompose-and-join framework to solve subgraph enumeration
- TwinTwigJoin = Simple graph storage (twintwigs as the join units) + Optimal left-deep join
- SEED = SCP graph storage (star and clique as the join units) + Optimal bushy join

Q & A

THANK YOU!